International Journal of Thermal Sciences 48 (2009) 1259–1264

Contents lists available at ScienceDirect

International Journal of Thermal Sciences

www.elsevier.com/locate/ijts

Thermal transport of fluid containing homogeneous microstructures

Debapriya Chakraborty, Suman Chakraborty *

Department of Mechanical Engineering, Indian Institute of Technology, Kharagpur 721302, India

ARTICLE INFO

Article history:
Received 31 July 2008
Received in revised form 5 November 2008
Accepted 8 December 2008
Available online 6 January 2009

Keywords:
Microstructures
Micropolar
Nusselt number
Micro-rotation slip

ABSTRACT

The present work develops a unified transport model towards investigating the thermal transport for fluids containing homogeneous and freely rotating internal structures in conduits with dimensions comparable to the structures present. In the continuum limit, such fluids are modeled using the micropolar theory. Utilizing a generalized micro-rotation slip boundary condition, closed form expressions are derived for the Nusselt number variations for thermally fully developed flows. These results provide valuable physical insights on the implications of the micro-rotation slip parameter and the viscosity ratios on the convective heat transfer characteristics.

1. Introduction

Numerous experimental and theoretical studies have been reported in the literature for analyzing the pertinent transport processes, it has off late been recognized that the classical Navier–Stokes (CNS) theory, describing the flow of a homogeneous, isotropic Newtonian fluid in a continuum, might turn out to be incomplete [1,2] in predicting the convective flow physics of fluids containing homogeneous or heterogeneous structured particles. One of the key reasons behind this apparent anomaly lies in the fact that the micro-rotational effects due to angular motion of molecules are neglected in the CNS theory, which might turn out to be of profound importance for analyzing the flows of particle-laden fluids or polymeric suspensions. In particular, deployment of the CNS theory with implicit ignorance of the underlying internal structure of fluid may lead to erroneous conclusions [3–6] for fluids with characteristic morphological features. For instance, although in vicinity of the channel centerline the dynamics of fluids containing structured particles may be satisfactorily explained through the classical Navier–Stokes (CNS) model, the discrepancies between the actual behavior and the CNS model based predictions tend to become progressively more ominous as one approaches towards the walls of the confinement [3–7]. The micropolar fluid theory, first proposed by Eringen [8–10], has proven to be a good substitute for such classes of fluids. This approach of modifying the classical theory is usually implemented by introducing higher order kinematic variables to impart higher degree of freedom. Micropolar fluids, containing rigid particles in a small volume, are capable of rotating about the centre of the volume element, as described by a micro-rotation vector. This local rotation of the particles is considered to be in addition to the usual rigid body motion of the entire volume element. The laws of classical continuum mechanics are extended in the micropolar theory with additional equations, which account for the conservation of micro-inertia moments and the balance of first stress moments that arise due to consideration of microstructures in a material. The micropolar fluid theory, thus, can support couple stresses and body couples, unlike the CNS model.

Despite the enormous promises offered by the micropolar fluid theory in analyzing convective transport issues of morphologically complicated fluids, there have been several bottlenecks against a consistent use of this theory for solving involved physical problems in reality. One important reason behind such practical constraints has been the frequent ill-handling of the additional equation for angular momentum conservation appearing in the micropolar theory, resulting in intrinsic inabilities of the resultant simplified sets of equations in capturing new physics and in bringing out valuable fundamental insights through the simulation predictions. Because of the non-trivial physical consequences of the additional variables introduced through the pertinent sets of governing equations, any prior experience in handling the CNS is not likely to be much helpful in this respect in translating and extending simple physical concepts into equivalent mathematical statements. This issue is further aggravated by a strong sensitivity of the resulting velocity and temperature profiles on the concerned phenomenological coefficients, even in the hydrodynamically and thermally fully developed limits. The most important constraints, in this respect, perhaps originate from the possible arbitrary, non-generic, and unphysical imposition of the micro-rotation boundary conditions in order to close the underlying mathematical model. As an obvious conclusion, there are every reasons to infer that the major complications

* Corresponding author. Tel.: +91 3222 282 990; fax: +91 3222 282 278.
E-mail address: suman@mech.iitkgp.ernet.in (S. Chakraborty).

1290-0729/$ – see front matter
doi:10.1016/j.ijthermalsci.2008.12.003

arising with the implementation of the micropolar fluid theory do not fundamentally originate as a consequence of the complexities in the transport equations themselves, but may primarily be attributed to the fact that for a given problem, several combinations of physically acceptable phenomenological coefficients and boundary conditions are mathematically possible and permissible. However, each of these combinations is capable of producing solutions that differ markedly from one another. As such, even with a fixed set of phenomenological coefficients, drastically contrasting velocity and temperature profiles can result from the application of slightly different boundary conditions [11]. Such anomalous predictive possibilities, however, have no counterparts in the CNS.

Despite being aware of the non-trivialities associated with the imposition of micro-rotation wall boundary condition [12], researchers in most cases have made controversial choices of the same in solving challenging physical problems. For example, many researchers prefer in specifying the wall micro-rotation parameter to be either zero or half the vorticity vector [13–18] at the solid boundaries, which have marginal physical basis and are only correct for limited physical situations. A generalized treatment in adjusting the diverse micro-rotational boundary conditions, so as to fit a variety of physical phenomena in a single unified framework, is therefore likely to be one of the key contributions towards the development of a comprehensive convective heat transfer theory for a general class of micropolar fluids. Aim of the present study, accordingly, is to devise a comprehensive convective heat transfer model for microchannel transport of micropolar fluids, with the aid of a unified micro-rotation slip boundary condition. The generalized proposition, in its asymptotic limits, is designed to be capable of accommodating all types of physically plausible micro-rotational boundary conditions within a linearized constitutive framework. Analytical expressions for the Nusselt number, consistent with this theory, are derived for constant wall flux boundary conditions, so as to impart valuable physical insights towards a comprehensive understanding of the implication of the internally rotating structures of the micropolar fluids on the effective thermal transport characteristics.

2. Fundamental model considerations for fluid flow and generalization of the approach

In micropolar fluids, the rigid particles contained in infinitesimal volume elements can independently rotate with respect to the center of the volume element, as quantified by the micro-rotation vector ($\boldsymbol{\omega}$), in addition to motion of the entire volume element described by the velocity vector ($\mathbf{V}$). In micropolar fluid theory, therefore, additional considerations need to be invoked to account for the conservation of micro-inertia moments and the first stress moments that arise from the contributions of the internal microstructure of the fluent medium. Thus, additional kinematic variables such as the gyration tensor and the micro-inertia moment tensor are introduced, and the concepts of body moments, stress moments, and micro-stresses are embedded in the micropolar theory, to arrive at the extended sets of continuum conservation equations.

For an incompressible fluid with constant material coefficients and with decoupled fluid flow and energy conservation equations, as well as with no external body forces or body torques, the equations for continuity, balance of momentum, and conservation of micro-inertia moments can be expressed as [8–10]

$$\nabla.\mathbf{V}=0 \tag{1}$$

$$\rho\frac{D\mathbf{V}}{Dt}=-\nabla p-(\mu+\kappa)\nabla\times\nabla\times\mathbf{V}+\kappa\nabla\times\boldsymbol{\omega} \tag{2}$$

$$\rho\frac{D\boldsymbol{\omega}}{Dt}=(\alpha+\eta+\gamma)\nabla(\nabla.\boldsymbol{\omega})-\gamma\nabla\times\nabla\times\boldsymbol{\omega}+\kappa\nabla\times\mathbf{V}-2\kappa\boldsymbol{\omega} \tag{3}$$

where p is the hydrostatic pressure; μ, κ are the coefficients of shear and vortex viscosities respectively; α, η, γ are the coupled viscosities. The governing equations described as above are suitably non-dimensionalized by employing the following scales: length scale $\to L_c$ (an intrinsic characteristic length scale of flow, taken to be equal to $\sqrt{\gamma/\kappa}$), velocity scale $\to U$ (a characteristic flow velocity, depending on the specific nature of flow), micro-rotation scale $\to U/L_c$, and pressure gradient scale $\to U(\mu+\kappa)/L_c$. It may be noted that the micropolar particles are associated with an intrinsic characteristic length scale of L_c, given by $L_c\sim\sqrt{\gamma/\kappa}$, which is typically of the order of few nanometers [19,20]. It may therefore be inferred that the micropolar theory becomes progressively more important when the characteristic system length scale tends to become comparable with the order of L_c. Thus, for microchannels and nanochannels, the implications of considering the micropolar effects may turn out to be immensely critical towards an accurate prediction of the underlying thermo-fluidic transport characteristics. However, for larger-sized systems (typically, the systems for which these two length scales are disparate by an order of 10^4 or more), predictions from the micropolar theory and the CNS model tend to converge on one another, except for regions infinitesimally close to the channel walls.

One may simplify the above governing equations further by utilizing the fact that microchannel flows are typically associated with low Reynolds numbers, so that under steady state conditions Eqs. (1)–(3) can be re-written as

$$-\nabla p+\nabla^2\mathbf{V}+\frac{\kappa}{\mu+\kappa}\nabla\times\boldsymbol{\omega}=0 \tag{4}$$

$$\nabla^2\boldsymbol{\omega}+\frac{\kappa}{\gamma}L_c^2(\nabla\times\mathbf{V}-2\boldsymbol{\omega})=0 \tag{5}$$

Eqs. (4), (5) can be simplified further for a two-dimensional, fully developed microchannel flow, for which one may write $\mathbf{V}=(u(y),0,0)$ and $\boldsymbol{\omega}=(0,0,\omega(y))$. Solution of the resultant coupled set of simplified governing equations requires four independent boundary conditions to be specified, with one half of the microchannel being considered as the problem domain. Two boundary conditions on velocity are relatively straight forward and trivial, in this respect. Further, the value of ω at the channel centreline can be taken as zero, because of a skew-symmetric nature of the micro-rotation tensor. However, the fourth boundary condition, describing the micro-rotation at the channel walls, cannot be trivially imposed without invoking further considerations. Diverse versions of this fourth boundary condition have routinely been proposed in literature [11,12,21,22], which are applicable only under restrictive constraints. For example, $\omega=0$ at the walls represents only a special case of concentrated particle flows, in which the microelements close to the wall surface are unable to rotate. Another type of micro-rotation wall boundary condition that has been commonly employed is of the form [17,21] $\omega|_{\text{wall}}=n\frac{\partial u}{\partial y}|_{\text{wall}}$, where n is an index that lies between 0 and 1. One possible argument in favor of this boundary condition is the fact that in the neighborhood of the wall, the only rotation is due to fluid shear and therefore the gyration vector must be equal to fluid vorticity, which characterizes weak concentrations. However, it is worth noting that $n=\frac{1}{2}$ indicates the vanishing of anti-symmetrical part of the stress tensor hence the necessity of higher order modeling can be questioned. Alternatively, the value of the couple stress at the wall, in form of the normal gradient of ω, may also be specified. Such diverse and restrictive options of the micro-rotation wall boundary condition make it an immensely difficult and challenging proposition to arrive at a generic transport model for micropolar fluids, with an inherent capability of mimicking a wide variety of near-wall considerations in a unified mathematical framework.

In order to overcome the above constraint, a generalized micro-rotation slip boundary condition is adopted in this work, with regard to the variation of ω at the microchannel walls. With this consideration, the complete set of boundary conditions can be described as

$$u = 0 \quad \text{at the walls } (y = \pm 1) \tag{6}$$

$$du/dy = 0 \quad \text{at the centreline } (y = 0) \tag{7}$$

$$\omega = 0 \quad \text{at the centreline } (y = 0) \tag{8}$$

$$\omega|_w = \beta \frac{d\omega}{dy}\bigg|_w \tag{9}$$

where β is the non-dimensional coefficient representing the micro-rotation slip length and subscript w denoting walls. For prescribing the velocity boundary condition at the walls, a no-slip boundary condition is assumed here for simplicity, with an understanding that more general considerations (such as apparent velocity slip due to hydrophobic interactions) can elegantly be accommodated by introducing a velocity slip coefficient or slip length, without disturbing the other generic aspects of the model formulation. Generalization of the velocity boundary condition, not being an objective of the present study and being extensively discussed in the reported literature, is not considered here. Rather, specific attention is focused in this study towards the generalization of the micro-rotation wall boundary condition in an effort to encapsulate all possible physical considerations in the underlying mathematical framework, and its consequent manifestation in the heat transfer characteristics. It is important to mention in this context that the no slip condition as given in Eq. (6) for a structured fluid ensures the following parameters to be zero at the stationary wall: (a) the velocity of the bulk fluid and (b) an intrinsic velocity of a part of a structure induced by its micro-rotation. Although the intrinsic velocity does not occur in any of the differential equations governing the mathematical theory of structured fluids, the micro-rotation occurs to be an important variable appearing in these equations. It has been classically hypothesized that the intrinsic velocity may be obtained from the micro-rotation through kinematic constitutive equations and an intrinsic displacement vector. Since these equations are linear and homogeneous at the boundary, it follows that if the micro-rotation vanishes at a boundary then the intrinsic velocity vanishes, too. However, there is no physical reasoning to substantiate that the converse is also true. Thus, the imposition of the no-slip boundary condition at the walls for the velocity of the bulk fluid mentioned as above does not constrain the micro-rotations to be zero at the wall.

One significant mathematical motivation towards adopting the generic form of the micro-rotation wall boundary condition given by Eq. (9) in that respect is that it is essentially of mixed type, and thereby can be asymptotically adjusted to Dirichlet and Neumann type of boundary conditions in appropriate physical limits. Eq. (9) also takes into consideration that because of a deviation from an ideally 'concentrated' state, the value of ω at the wall may deviate from zero. The extent of this deviation, in a linear constitutive framework, is represented by the micro-rotation slip parameter β, which physically represents a characteristic non-dimensional distance over which the near-wall gradients in ω are estimated to be operative. Zero couple stress at the wall represents a limiting case of this situation, for which the above length scale tends to infinity so as to render a finite value of ω at the microchannel walls. β can take positive as well as negative values depending on the sense of micro-rotation gradient at the wall.

Based on the boundary conditions given by Eqs. (6)–(9), closed form expressions for the velocity and the micro-rotation parameters can be obtained as

$$\omega(y) = \frac{(1-\beta)\sinh(Ny)}{\sinh(N) - \beta N\cosh(N)}\frac{1}{N^2} - \frac{y}{N^2} \tag{10}$$

$$u(y) = \frac{(1-y^2)}{N^2} + \frac{\kappa}{2\mu+\kappa}\frac{(1-\beta)}{N[\sinh(N) - \beta N\cosh(N)]} \times \left[\cosh(Ny) - \cosh(N)\right] \tag{11}$$

where $N = \sqrt{(2\mu+\kappa)/(\mu+\kappa)}$. Eq. (11), in a special case, mimics the classical Poiseullean velocity profile in absence of any micro-rotational effects, as derived by imposing $\kappa = 0$ (equivalently, $N = \sqrt{2}$).

The value of the micro-rotation slip coefficient, β, specific to a given physical situation, can be obtained in principle by executing molecular dynamics calculations for a reduced flow domain in the form of a wall-adjacent layer of length scale L_c, and conceptualizing the flow occurring across the same to be equivalent to a local-scale Couette–Poiseuille flow close to the channel walls [23]. This is in accordance with the physical consideration that fluid flow over the solid walls can be conceived as an equivalent local-scale combined shear- and pressure-driven flow between two confining boundaries separated by a narrow gap of L_c, with the bottom boundary being at rest and top boundary translating with an arbitrary non-dimensionalized velocity, u_0. To the extent that the wall-adjacent fluid layer is adequately resolved, u_0 here may act as a free parameter that can be tuned with the help of local-scale molecular dynamics simulation predictions.

It is important to recognize here that the micro-rotation slip parameter, β, acts as one of the most significant factors in dictating the transport phenomena within the microchannel. By referring to extensive simulation studies on combined shear- and pressure-driven flows (Couette–Poiseuille flows) [23] in which the bottom plate is at rest and the top plate moves with a uniform axial velocity u_0, it is interesting to observe that the cases with different values of β follow a kind of universal trend, which can be mathematically expressed as

$$\beta - \beta_0 = \chi\left(\left|\frac{d\omega}{dy}\right|_w\right)^{-1} \tag{12}$$

where $\chi = 0.5u_0 + 0.036255$ and β_0 represents an appropriate shift parameter which can be represented as a function of u_0 as

$$\beta_0 = \frac{-1.174u_0^2 + 0.1341u_0 - 0.002824}{u_0^2 - 0.1588u_0 + 0.006256} \tag{13}$$

A motivation of introducing the parameter u_0 here lies in the fact that the micropolar considerations for microchannel flows are likely to become significant only in a wall-adjacent layer of length scale L_c, within which the flow can locally be modeled as a combined pressure- and shear-driven one, as mentioned earlier. With molecular dynamics simulations resolving only up to that length scale, the value of u_0 solely takes the burden of transmitting the consequences of the micro-rotational effects from the wall-adjacent layers to the outer layers, without necessitating any resolution of the remaining portion of the domain upto the molecular scales. This, in turn, can completely prescribe the micro-rotation wall boundary condition as per Eq. (12) without any ambiguity, through the pertinent micro-rotation slip parameter. A standardization of this computationally efficient multi-scale approach is detailed in [23], and is not repeated here for the sake of brevity.

A comparison between the velocity profiles obtained from using Eq. (11) and the predictions from the CNS theory is demonstrated in Fig. 1, for $\beta = -3, -1, 0, 1, 3$. Since the boundary condition constrains the values of u to be zero at walls, there is no difference in the velocity profiles predicted by the two theories at the walls. However, it can be observed that the micro-rotation effects enhance the linear velocities for all values of the slip parameter, β. Further, it is interesting to note that the maximum volume flow rate is obtained for $\beta = 0$. This can be attributed to the maximum possible strength of the micro-rotation gradient that locally prevails in the wall-adjacent layer under these conditions, which can

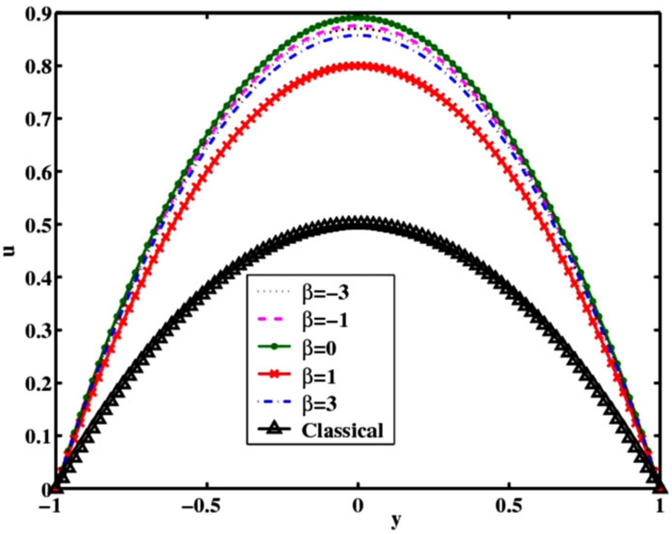

Fig. 1. Profiles of u vs. y using classical theory and micropolar theory with $\kappa/\mu = 3$ and $\beta = -3, -1, 0, 1, 3$.

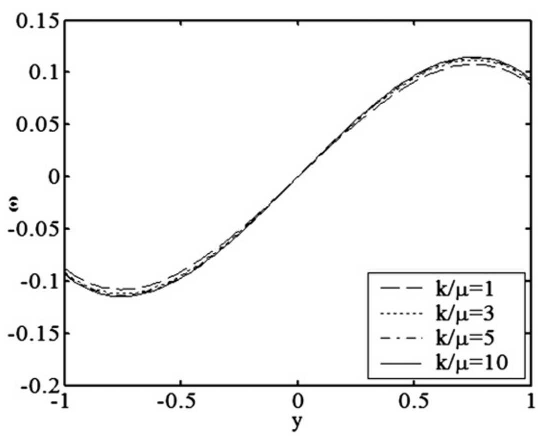

Fig. 2. Profiles of ω vs. y for different values of κ/μ and $\beta = -0.5$.

efficiently impart kinetic energies into the outer flow in the most effective manner. Because of the anti-symmetries in the values of ω over the two geometrically similar half spaces constituting the microchannel, it is primarily the magnitude of β and not its sign that dictates the velocity profiles and flow directions, as evident from Fig. 1.

A typical fully developed profile for micro-rotation is depicted in Fig. 2, with varying $\frac{\kappa}{\mu}$, for $\beta = -0.5$. It can be observed that the profile is anti-symmetric about the centreline. Although ω assumes non-zero values at the walls in general, special cases of strong concentrations or strong adherences to the wall can also be handled by the theory, by setting $\beta = 0$.

3. Thermal transport

The governing differential equation for energy conservation at steady state, under the simplified assumptions of constant physical properties with negligible viscous dissipation effects, can be stated as

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right) \tag{14}$$

where T is the temperature, and α is the thermal diffusivity. Further simplifications in Eq. (14) can be made by imposing specific boundary conditions at the channel walls. Here, we consider the constant wall heat flux (q_w) boundary condition, which in conjunction with a thermally fully developed constraint [24] yields

$$\frac{\partial T}{\partial x} = \frac{dT_m}{dx} = \frac{dT_w}{dx} = \text{constant} \tag{15}$$

where T_w is the wall temperature and T_m is the bulk mean flow temperature.

Using the velocity distribution given by Eq. (11), the temperature distribution for the constant heat flux boundary condition can be obtained as

$$T = T_w + Pe\frac{dT_m}{dx}\left[\frac{1}{N^2}\left(\frac{y^2}{2} - \frac{y^4}{4} - \frac{5}{12}\right) + B\left\{\frac{\cosh(Ny)}{N^2} - \left(\frac{y^2}{2} - \frac{1}{N^2} + \frac{1}{2}\right) \times \cosh(N)\right\}\right] \tag{16}$$

where

$$B = \frac{(1-\beta)}{\left(-2\frac{1-N^2}{2-N^2} + 1\right)N[\sinh(N) - \beta N\cosh(N)]} \tag{17}$$

In Eq. (16), Pe is the thermal Peclet number of flow. Again, the special cases for standard fluids can be retrieved from Eq. (16), by setting $\kappa = 0$ (i.e., $N = \sqrt{2}$), which recovers the classical theory based standard heat transfer predictions for thermally fully developed internal flows.

Further, by utilizing the following definition of T_m:

$$T_m = \frac{\int_{-1}^{+1} uT\,dA}{\int_{-1}^{+1} u\,dA} \tag{18}$$

Eq. (16) yields

$$T_m = T_w + A\frac{dT_m}{dx}\frac{Pe}{2u_{av}} \tag{19}$$

where the parameter A can be expressed as

$$A = -\frac{1}{315N^7}\big[-315B^2N^5 + \{84BN^5 - 420B^2N^7 - 2520BN\}\cosh(N) + 2520B\sinh(N) + \{315B^2N^4 + 630B^2N^6\}\cosh(N)\sinh(N) + 136N^3\big] \tag{20}$$

and u_{av} is the average velocity calculated from Eq. (11), and is given by

$$u_{av} = \frac{2}{3N^2} + \frac{B}{N}\big[\sinh(N) - N\cosh(N)\big] \tag{21}$$

The temperature can be non-dimensionalized in the form of

$$\theta = \frac{T - T_w}{T_m - T_w} \tag{22}$$

Utilizing the wall boundary condition:

$$-k\frac{dT}{dy}\bigg|_{y=1} = h(T_m - T_w) \tag{23}$$

the expression for the Nusselt number (Nu) can be obtained from the basic definition $Nu = \frac{hD}{k}$ (D being the hydraulic diameter of the channel) as

$$Nu = \frac{d\theta}{dy}\bigg|_{y=1} = 8u_{av}\frac{2/3N^2 + B\left(\frac{\sinh(N)}{N} - \cosh(N)\right)}{A} \tag{24}$$

It is important to observe here that the Nusselt number variation depends on two important micropolar parameters, namely, N (a function of $\frac{\kappa}{\mu}$) and β. The variation of Nu with changes in the parameter N is plotted in Fig. 3. It can be observed that for $\kappa = 0$ (i.e., $N = \sqrt{2}$) the value of $Nu = 8.2353$, which matches exactly with the value obtained from the classical heat transfer theory. In

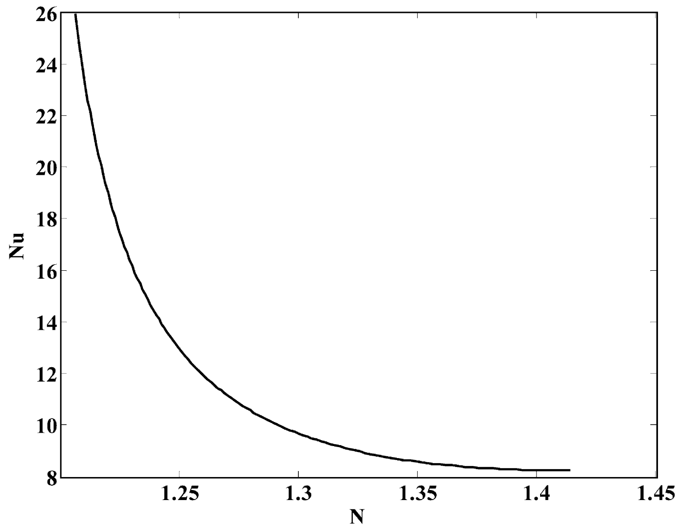

Fig. 3. Variation of the Nusselt number with the parameter N with $\beta = -0.5796$ (a value consistent with the outcome of molecular dynamics simulations [23] of a benchmark problem).

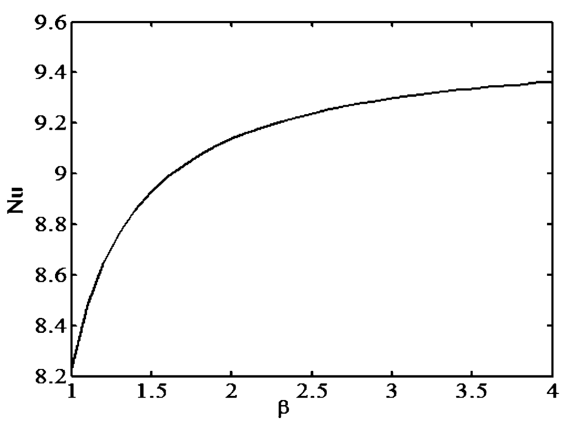

Fig. 4. Variation of Nu vs. β with $N = 1.2910$ corresponding to $\kappa/\mu = 0.5$.

general, the variation of Nu can be observed to be a function of the rheological constants of the fluid. When the ratio between the viscosity coefficient and the shear viscosity coefficient increases, the parameter N also increases and the Nu tends to decrease. The shear viscosity effects, thus, tend to aid the convective heat transfer, by interacting with the micro-rotational elements, so that heat can efficiently get transmitted from one representative elemental volume of fluid to another.

The variations in the Nu with the parameter β are depicted in Fig. 4. It can be observed from the figure that the Nu tends to increase with increases in the value of β. This is in accordance with the augmentations in the convective flow velocities with enhancements in magnitude of the parameter β, as evident from Fig. 1, corresponding to its non-zero values. With higher values of β, the wall-adjacent micro-rotational elements transport thermal energy more efficiently from the walls into the bulk of the fluid, as aided by the combined effects of conduction and advection. With high values of β, however, a limiting asymptotic state is also reached, which represents the physical situation corresponding to vanishingly small couple stress values close to the walls. Such non-trivial and diverse physical implications may become important in predicting the convective heat transfer characteristics for the fluids whose characteristic dimension of internal structures, L_c, is comparable to the characteristic length scales of flow.

4. Conclusions

In the present work, the micropolar fluid theory has been explored to analyze the convective transport features of flows with homogeneous rotating internal structures. Towards achieving this objective, a generalized micro-rotation boundary condition has been considered at the microchannel walls, as expressed in the form of a micro-rotation slip parameter. This approach provides a sound basis for modeling rheologically complex flows through small channels in a mathematically unified framework, instead of using complicated and restricted empirical relationships for the constitutive modeling. As such, the effective generalization of the micro-rotation slip condition offers with an excellent flexibility of representing the underlined flow physics with simple linearized constitutive relationships for the couple stresses. Convective heat transfer analysis is subsequently carried out, based on this generic proposition. It is demonstrated that the internal variables influence the temperature variations in a micropolar fluid to a rather significant extent. The Nusselt number variation is also obtained as a closed form solution, for thermally fully developed flows with constant wall heat fluxes. It is inferred that the ratio between the vortex viscosity coefficient and the shear viscosity coefficient, along with the micro-rotation slip parameter, bear profound consequences with regard to their implications on the microchannel heat transfer rates, and can possibly be exploited to devise novel heat transfer augmentation strategies in systems of microscopic length scales.

References

[1] K.P. Travis, B.D. Todd, D.J. Evans, Departure from Navier–Stokes hydrodynamics in confined liquids, Phys. Rev. E 55 (4) (1997) 4288–4295.
[2] J. Delhommelle, D.J. Evans, Poiseuille flow of a micropolar fluid, Molecular Physics 100 (17) (2002) 2857–2865.
[3] P. Tabeling, Some basic problems of microfluidics, in: 14th Australian Fluid Mech. Conference, Dec. 2001, pp. 10–14.
[4] K.V. Sharp, R.J. Adrian, J.G. Santiago, J.I. Molho, Liquid flows in micro-channels, University of Illinois, Urbana-Champaign, TAM Report No. 961, Nov. 2000.
[5] X.F. Peng, B.X. Wang, Liquid flow and heat transfer in microchannels with/without phase change, in: Proc. 10th International Heat Transfer Conference, Brighton, UK, Aug. 14–18, 1994, pp. 159–177.
[6] D.B. Holmes, J.R. Vermeulen, Velocity profiles in ducts with rectangular cross sections, Chem. Eng. Sci. 23 (1968) 717–722.
[7] K.P. Travis, B.D. Todd, D.J. Evans, Poiseuille flow of molecular fluids, Physica A 240 (1997) 315–327.
[8] A.C. Eringen, Simple microfluids, Int. J. Eng. Sci. 2 (1964) 205–217.
[9] A.C. Eringen, E.S. Suhubi, Nonlinear theory of simple micro-elastic solids, Int. J. Eng. Sci. 2 (1964) 89–203.
[10] A.C. Eringen, Theory of micropolar fluids, J. Math. Mech. 16 (1966) 1–18.
[11] A.D. Kirwan Jr., N. Newman, Plane flow of a fluid containing rigid structures, Int. J. Eng. Sci. 7 (1969) 883.
[12] A.D. Kirwan Jr., Boundary conditions for micropolar fluids, Applied Engg. Sci. 24 (7) (1986) 1237–1242.
[13] O. Aydin, I. Pop, Natural convection in a differentially heated enclosure filled with a micropolar fluid, International Journal of Thermal Science 46 (2007) 963–969.
[14] C.L. Chang, Buoyancy and wall conduction effects on forced convection of micropolar fluid flow along a vertical slender hollow circular cylinder, International Journal of Heat and Mass Transfer 49 (25–26) (2006) 4932–4942.
[15] F.S. Ibrahim, I.A. Hassanien, Mixed convection boundary layer flow of a micropolar fluid on a horizontal flat plate with power law variation in surface temperature, International Journal of Thermal Science 39 (2000) 360–373.
[16] C.Y. Cheng, Fully developed natural convection heat and mass transfer of a micropolar fluid in a vertical channel with asymmetric wall temperatures and concentrations, International Communications in Heat and Mass Transfer 33 (5) (2006) 627–635.
[17] A.J. Chamka, T. Grosan, I. Pop, Fully developed convection of micropolar fluid in a vertical channel, Int. Comm. Heat Mass Transfer 29 (8) (2002) 1119–1127.
[18] C.Y. Cheng, Free convection heat and mass transfer from a horizontal cylinder of elliptic cross section in micropolar fluids, Int. Comm. Heat Mass Transfer 33 (3) (2006) 311–318.

[19] J. Delhommelle, D.J. Evans, Configurational temperature thermostat for fluids undergoing shear flow: application to liquid chlorine, Molecular Phys. 99 (2001) 1825–1829.
[20] J. Delhommelle, Rotational viscosity of uniaxial molecules, Molecular Phys. 100 (2002) 3479–3482.
[21] G. Ahmadi, Self-Similar Solution of incompressible micropolar boundary layer flow over a semi-infinite plate, Int. J. Eng. Sci. 14 (1976) 639–646.
[22] J. Peddieson Jr., An application of the micropolar fluid model to the calculation of turbulent shear flow, Int. J. Eng. Sci. 23 (10) (1972) 23–32.
[23] D. Chakraborty, S. Chakraborty, Towards a generalization of boundary conditions for fluids with homogeneous internally rotating structures, Phys. Lett. A 372 (33) (2008) 5462–5466.
[24] W.M. Kays, M.E. Crawford, Convective Heat and Mass Transfer, third ed., McGraw-Hill Book, New York, 1993.